# Solutions to JEE MAIN-2|JEE 2024

## PHYSICS

## SECTION-1

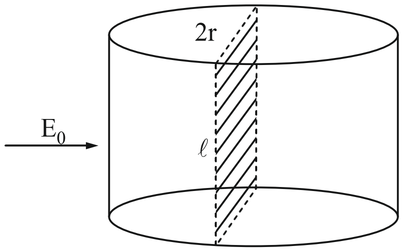
**1.(C)** For uniformly charged circular arc

$E_0 = \dfrac{2k\lambda}{R}\sin\alpha$, where $\alpha$ is half angle it forms at centre

For 1, $\alpha = 135°$, $E_0 = \sqrt{2}\dfrac{k\lambda}{R}$

For 2, $\alpha = 45°$, $E_0 = \sqrt{2}\dfrac{k\lambda}{R}$

**2.(D)** $\phi = E_0$ . Projected Area




$= E_0 (2r\ell)$

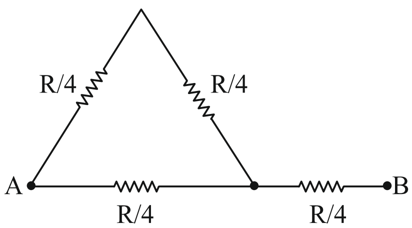
**3.(B)** Due to upper half and lower half ring potential will be equal but of opposite sign, while electric field will be equal and in same direction.

**4.(D)** $R = \dfrac{\rho\ell}{A}$ ; $x = \dfrac{\rho .3a}{2a^2}$

$y = \dfrac{\rho \cdot 2a}{3a^2}$ ; $z = \dfrac{\rho \cdot a}{6a^2}$

**5.(A)** $R_{AB} = \dfrac{2}{3}.\dfrac{R}{4} + \dfrac{R}{4}$

$R_{AB} = \dfrac{5}{12}R$




**6.(B)** As $R_A > R_B$

Before closing switch, $i$ = const.

$V_A > V_B$

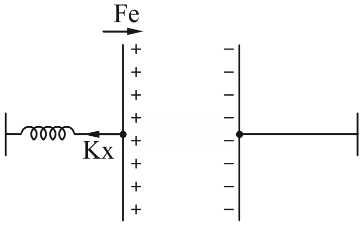
After closing switch, $V_A = V_B$

As potential differecne across A decreases, Intensity decreases

**7.(D)** Charge stored in $5\mu f = 0$ due to short circuit

So energy = 0

**8.(C)** $Kx = F_e$




$$Kx = \frac{q^2}{2A\varepsilon_0} \quad \Rightarrow \quad x = \frac{q^2}{2K\varepsilon_0 A}$$

**9.(C)**

**10.(B)** By kirchhoff's junction law

$$i_1 = i_2 + i_3$$

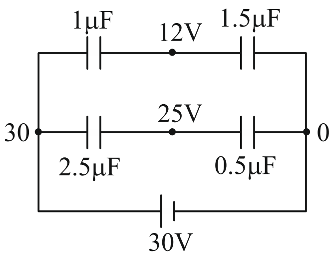
$$\frac{50-V}{5} = \frac{V-0}{3} + \frac{V-30}{15}$$

$$150 - 3V = 5V + V - 30$$

$$9V = 180 \ ; \ V = 20V$$

$$i_1 = \frac{30}{5} = 6A, \ i_2 = \frac{20}{3}A, i_3 = \frac{10-30}{15} = \frac{2}{3}A$$

**11.(D)**



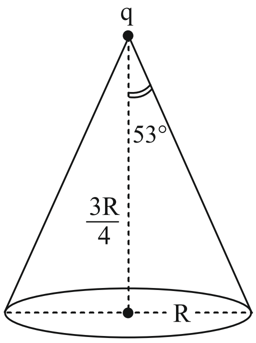

**12.(A)** $J = \frac{I}{A} \Rightarrow J_A > J_B$ (as I is same)

$J = \sigma E \Rightarrow E_A > E_B$ (as $\sigma$ is same)

**13.(B)** $\phi = \frac{q}{2\varepsilon_0}(1 - \cos 53°)$

$$= \frac{q}{2\varepsilon_0}\left(1 - \frac{3}{5}\right)$$

$$= \frac{q}{5\varepsilon_0}$$

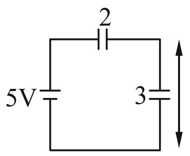



**14.(D)** Resistance of voltmeter is infinity. So current drawn from battery is zero.

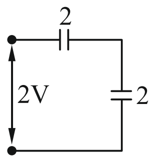
$\therefore \quad I = \frac{V}{2R}$

P.d across $R = IR = \frac{V}{2}$

**15.(C)**




$$\frac{2}{2+3}\times 5 = 2V$$

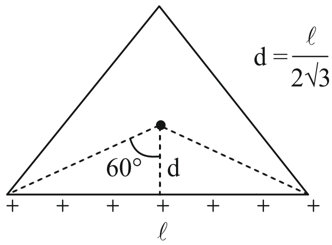



$\therefore \quad 1V$

**16.(D)** $E_0 = \dfrac{2k\lambda}{d}\sin\theta$

$$= \frac{2k\lambda}{\left(\frac{\ell}{2\sqrt{3}}\right)}\cdot \sin 60^\circ$$

$$= \frac{6k\lambda}{\ell} = \frac{6kq}{\ell^2}$$




**17.(B)** Current through battery

$$i = \frac{1.5+1.5}{\left(\frac{15\times 10}{15+10}+2+2r\right)} \qquad \therefore \quad \text{Current through } 10\Omega \text{ resistor;}$$

$$i_{10\Omega} = \frac{15}{15+10}\times i = \frac{3}{5}\times i \qquad \therefore \quad \text{voltage across } 10\Omega \text{ resistor} = \left(\frac{3}{5}i\right)\times 10 = 2 \text{ (Given)}$$

$$\Rightarrow \quad \frac{3}{5}\times\frac{3\times 10}{(6+2+2r)} = 2 \quad \Rightarrow \quad (8+2r) = \frac{90}{10} = 9 \quad \Rightarrow \quad r = \frac{1}{2} = 0.5\Omega$$

**18.(B)** $V_R = \varepsilon . e^{-t/RC}$

$V_C = \varepsilon[1-e^{-t/RC}]$

At $t = 100\,ms$

$$V_R = V_C \quad \Rightarrow \quad e^{-t/RC} = \frac{1}{2} \;;\; RC = \frac{100}{\ln(2)} \approx 145.45\,ms$$

**19.(B)** $q_{flown} = CV - (-2CV) = 3CV$

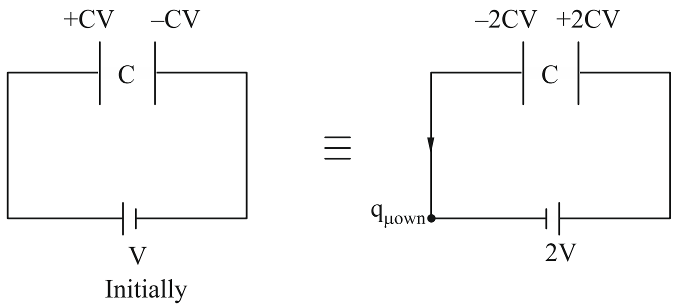
$$U_f = \frac{1}{2}C\times(2V)^2 = 2CV^2$$

$$U_i = \frac{1}{2}CV^2$$

$$\Delta U = 2CV^2 - \frac{1}{2}CV^2 = \frac{3}{2}CV^2$$

$$W_b = q_{flow}\,2V = +3CV\times 2V = +6CV^2$$

$$H = W_b - \Delta U = +6CV^2 - \frac{3}{2}CV^2 = \frac{9}{2}CV^2$$




Now, $\dfrac{H}{U_f} = \dfrac{\frac{9}{2}CV^2}{2CV^2} = \dfrac{9}{4} = 2.25$

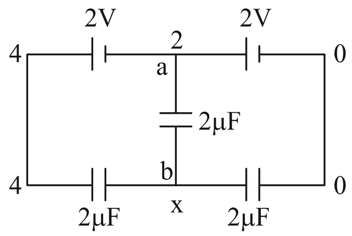
**20.(A)** KCL at junction

$2(x-0)+2(x-4)+2(x-2)=0$

$x=2$

$V_a=2, V_b=2$

$V_a-V_b=0$




## SECTION-2

**1.(110)** $dV=-Edx$

$\int_{5}^{V} dV=-\int_{2}^{5}(4-3x^2)dx$

$V-5=105$ ; $V=110$ volt

**2.(13)** $\rho=\frac{m}{ne^2\tau}$ ; $\tau=\frac{m}{\rho ne^2}=\frac{9.1\times10^{-31}}{2\times10^{-8}\times9\times10^{27}\times(1.6\times10^{-19})^2}=2\times10^{-13}$ sec

**3.(6)** $Q_{net}=1200\,pC$

After connecting the charge will divide equally as capacitance are same

$Q_1=Q_2=600\,pC$

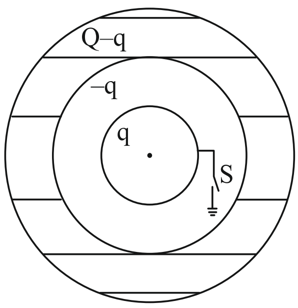
$H=\frac{1}{2}\frac{(1200)^2}{60}-2\times\frac{1}{2}\frac{(600)^2}{60}=6000\,pJ$ ; $H=6\,nJ$

**4.(4)** $\frac{k(q-Q)}{3R}-\frac{kq}{2R}+\frac{kq}{R}=0$

$\frac{kq-Q}{3R}=-\frac{kq}{2R}$

$q-Q=-\frac{3}{2}q$

$\frac{5}{2}q=Q$ ; $q=\frac{2}{5}Q$




**5.(3)** $I_g\cdot R_g=(I-I_g)R_{sh}$

$0.05\times50=(5-0.05).R_{sh}$

$R_{sh}=\frac{2.5}{4.95}$

$\rho\frac{\ell}{A}=\frac{50}{99}$

$\frac{5\times10^{-7}\times\ell}{2.97\times10^{-6}}=\frac{50}{99}$

$\ell=3m$

**6.(200)** In steady state current through battery $I=\frac{50}{23+3+17+7}=1A$

$\Rightarrow \quad Q_{10\mu F}=CV=10\times20=200\mu C$

**7.(180)** $\vec{r} = \overrightarrow{r_p} - \overrightarrow{r_0} = 6\hat{i} - 8\hat{j}$

$r = \sqrt{6^2 + 8^2} = 10m$

$V = k = \frac{q}{r} = 9 \times 10^9 \times \frac{200 \times 10^{-6}}{10}$

$= 180 \times 10^3$ Volt **;** $= 180\,kV$

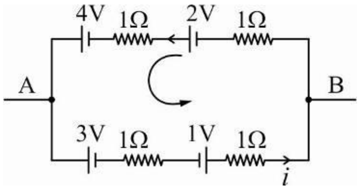
**8.(4)** From $R_t = R_0(1 + \alpha t)$ $\therefore$ $5 = R_0(1 + 50\alpha)$ ... (i)

and $6 = R_0(1 + 100\alpha)$ ... (ii)

$\therefore$ $\frac{5}{6} = \frac{1 + 50\alpha}{1 + 100\alpha}$ or $\alpha = \frac{1}{200}$

Putting the value of $\alpha$ in equation (i), we get: $5 = R_0\left(1 + 50 \times \frac{1}{200}\right) \Rightarrow R_0 = 4\Omega$

**9.(4)** Applying loop rule $I = \frac{2 + 4 - 3 + 1}{1 + 1 + 1 + 1} = 1\ amp$




$V_{AB} = 4 + 2 - 1 \times 2 = 4$ volts

**10.(4)** $V_{AC} = \frac{\ell}{L} \cdot V_{AB} = \frac{40}{100} \times 10 = 4V$

$i.R = 4$

$\frac{5}{1+R} \cdot R = 4$

$5R = 4 + 4R$

$R = 4\Omega$

# CHEMISTRY

## SECTION-1

**1. (B)** For zero order reaction, same amount of reactant is consumed in equal time interval.

| | | | | | |
|---|---|---|---|---|---|
| [X] | X | 0.75 X | 0.50 X | 0.25 X | 0 |
| Time | 0 | t | 2t | 3t | 4t |

Hence, $\left.\begin{array}{l} p = t \\ q = 2t \\ r = 3t \end{array}\right\}$

$\Rightarrow \quad t = p = \frac{q}{2} = \frac{r}{3} \Rightarrow 6p = 3q = 2r$

**2. (C)** Discharging tendency : $Ag^+ > Cu^{2+} > H^+ > Na^+$

| **Electrolyte** | **Product of electrolysis at cathode** |
|---|---|
| $CuSO_4(aq)$ | Cu(s) |
| $AgNO_3(aq)$ | Ag(s) |
| $NaCl(aq)$ | $H_2(g)$ |
| $Na_2SO_4(aq)$ | $H_2(g)$ |
| $HCl(aq)$ | $H_2(g)$ |

**3. (D)** Facts related to colloids

**4. (B)** $t_{1/2} \propto \frac{1}{(a)^{n-1}}$

[a = initial conc. of reactant]
[n = order w.r.t. reactant]

For A, $t_{1/2} \propto a$ [Given]

$\Rightarrow \quad t_{1/2} \propto \frac{1}{(a)^{-1}}$

Comparing, $n-1=-1$, $n=0$

$\rightarrow$ order w.r.t A = 0

For B, $t_{1/2} \times a = k$ [Given]

$t_{1/2} = \frac{k}{a} \Rightarrow t_{1/2} \propto \frac{1}{(a)^1}$

Comparing, $n-1=1$, $n=2$

Order w.r.t. B = 2

Overall order of reaction = 0 + 2 = 2

**5. (B)** $OH^-$ has maximum ionic molar conductivity due to grotthus conductance.

For alkali metal cations,

Hydrated Radius : $Li^+(aq) > Na^+(aq) > K^+(aq) > Rb^+(aq) > Cs^+(aq)$

Hydrated Radius $\uparrow$ Ionic mobility $\downarrow$ $\lambda_m \downarrow$

**6. (D)** Rate of reaction = $\dfrac{\text{Rate of disappearance / Appearance}}{\text{Stoichiometric coefficient}}$

$$\text{ROR} = \frac{(\text{ROD})_A}{2} = \frac{(\text{ROD})_B}{3} = \frac{(\text{ROA})_C}{1} = \frac{(\text{ROA})_D}{2}$$

$$= \frac{-1}{2}\frac{d[A]}{dt} = \frac{-1}{3}\frac{d[B]}{dt} = \frac{d[C]}{dt} = \frac{1}{2}\frac{d[D]}{dt}; \quad \frac{-2d[B]}{dt} = \frac{3d[D]}{dt}$$

**7. (B)** Facts related to enzymes.

**8. (C)** Facts related to Tyndall effects

**9. (D)** Nernst Equation for above cell reaction is

$$E_{cell} = E^{\circ}_{cell} \frac{-0.059}{5}\log\left\{\frac{[Mn^{2+}][Fe^{3+}]^5}{[MnO_4^-][H^+]^8[Fe^{2+}]^5}\right\}$$

pH $\downarrow$ $\quad$ $[H^+]$ $\uparrow$ $\quad$ Ecell $\uparrow$

**10. (B)** $\overset{\infty}{\lambda}_{eq}[Ca_3(PO_4)_2] = \overset{\infty}{\lambda}_{eq}[Ca^{2+}] + \overset{\infty}{\lambda}_{eq}[PO_4^{3-}] = x + y$

$\overset{\infty}{\lambda}_M = \overset{\infty}{\lambda}_{eq} \times n - \text{factor}$ ; $\overset{\infty}{\lambda}_M = (x+y)\times 6 = 6(x+y)$

**11. (A)** Rate = $k \cdot [A]^2[B]^1[C]^0$

$$k = \frac{\text{Rate}}{[A]^2[B]^1} = \frac{\text{Mol L}^{-1}\text{s}^{-1}}{(\text{Mol L}^{-1})^3} = \text{Mol}^{-2}\ \text{L}^2\ \text{s}^{-1}$$

**12. (B)** Use, $m^x = n$
m = No. of times a reactant concentration is made.
n = No. of times rate of reaction becomes
x = Order w.r.t. a reactant

| **For A** | **For B** |
|---|---|
| $m^x = n$ | $m^x = n$ |
| $2^x = 4$ | $\left(\frac{1}{2}\right)^x = \text{constant}$ |
| $2^x = 2^2$ | $(2)^{-x} = (2)^0$ |
| x = 2 | x = 0 |

Rate law expression is Rate $= k[A]^2[B]^0$

when volume is doubled, concentration of both the reactants is halved. Let new rate be $R^1$.

$$R^1 = k\left[\frac{1}{2}A\right]^2\left[\frac{1}{2}B\right]^0$$

$$R^1 = k \cdot \frac{1}{4} \cdot [A]^2 \times \left(\frac{1}{2}\right)^0 \cdot [B]^0$$

$$R^1 = \frac{1}{4} \cdot k \cdot [A]^2[B]^0; \ R^1 = \frac{1}{4}R$$

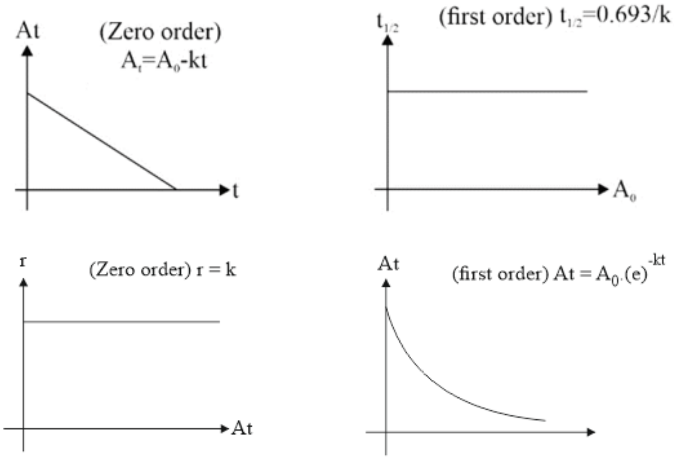
**13. (D)** $HClO_2 + 2H^+ + 2e^{\ominus} \longrightarrow HClO + H_2O$ ; $\Delta\ G_1^\circ = \left(-2FE_1^\circ\right)$ ...(i)

$2HClO + 2H^+ + 2e^- \longrightarrow Cl_2 + 2H_2O$ ; $\Delta G_2^\circ = -2FE_2^\circ$ ...(2)

$2HClO_2 + 6H^+ + 6e^- \longrightarrow Cl_2 + 4H_2O$ ; $\Delta G_3^\circ = -6FE_3^\circ$ ...(3)

$$\Delta \overset{\circ}{G}_3 = 2\Delta \overset{\circ}{G}_1 + \Delta \overset{\circ}{G}_2 \Rightarrow -6FE_3^\circ = -4FE_1^\circ - 2FE_2^\circ \Rightarrow E_3^\circ = \frac{\left(2E_1^\circ + E_2^\circ\right)}{3} = 1.659\ V$$

**14. (D)**




**15. (B)** Equivalents $= \dfrac{i \times t}{F}$. If same current is passed for same duration equivalents will be same. (1 : 1 : 1)

Equivalents = moles $\times$ n-factor.

If equivalent are same, moles $\propto \dfrac{1}{n - \text{factor}}$ n-factor $\rightarrow$ Ag : Cu : Au = 1 : 2 : 3

Moles $\rightarrow$ Ag : Cu : Au $= \dfrac{1}{1} : \dfrac{1}{2} : \dfrac{1}{3} = 6 : 3 : 2$

Moles deposited $\uparrow$ conc. Of ions in soln. $\downarrow$

**16.(A)** $\dfrac{x}{m} = kp^{\frac{1}{n}}$

$$\frac{0.2 = k4^{\frac{1}{n}}}{0.5 = k25^{\frac{1}{n}}}$$

$$\frac{2}{5} = \left(\frac{4}{25}\right)^{\frac{1}{n}}$$

$$\frac{2}{5} = \left(\frac{2}{5}\right)^{\frac{2}{n}}$$

$$1 = \frac{2}{n}$$

$$n = 2$$

Using n = 2

$$0.2 = k\ 4^{\frac{1}{2}}$$

$$\frac{0.2}{2} = k \qquad k = 0.1$$

At 36 bar

$$\frac{x}{m} = 0.1x\ \ 36^{\frac{1}{2}}$$

$$\frac{x}{m} = 0.6$$

Number of moles $= \dfrac{0.6}{28} = \dfrac{6}{28 \times 10} = \dfrac{3}{140} = 0.02$

**17. (A)** $k = \frac{2.303}{t}\log\left(\frac{A_0}{A_t}\right)$

$$k = \frac{2.303}{10}\log\left(\frac{2}{0.8}\right) \; ; \; k = \frac{2.303}{10}\times\log\left(\frac{5}{2}\right) = \frac{2.303}{10}\times 0.4 = 0.092 \text{ min}^{-1}$$

**18. (C)**

| | Anode | Cathode |
|---|---|---|
| Electrolytic Cell | + | - |
| Galvanic Cell | - | + |

**19. (B)** Gold sol is negative sol. It contains negatively charged $AuO_2^-$ ions adsorbed on it.

**20. (D)** Fact

## SECTION-2

**1. (54)** Conductance $(G) = \frac{1}{R} = \frac{1}{20} S$

Conductivity $(k)$ = conductance $(G)\times$ cell constant $(G^*)$

$$\text{Cell constant } (G^*) = \frac{k}{G} = \frac{0.027 \text{ S cm}^{-1}}{1/20 \text{ S}} = 0.54 \text{ cm}^{-1} = \cancel{Z} \quad ; \quad 100\cancel{Z} = 54$$

**2. (25)** By definition of gold number, the amount of protective colloid required to prevent coagulation of 10 ml gold sol against 1 ml 10% NaCl soln. = 50 mg

$$= 50\times 10^{-3} \text{ g}$$

For 10 ml gold sol, protective colloid required = $= 50\times 10^{-3}$ g

For 50 ml gold sol, protective colloid required = $5\times 50\times 10^{-3}$ g $= 0.25$ g

$x = 0.25$ g ; $100x = 25$

**3. (1)** $E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\log Q_c$ for concentration cell, $E^\circ_{cell} = 0$

$$E_{cell} = 0.12 = 0 - \frac{0.06}{2}\log\left\{\frac{1\times(0.001)^2}{1\times\left(10^{-x}\right)^2}\right\}$$

$$0.12 = -0.03\times\log\left\{\frac{10^{-6}}{10^{-2x}}\right\}$$

$$\Rightarrow \quad -\frac{0.12}{0.03} = \log 10^{-6} - \log(10)^{-2x} \Rightarrow \quad -4 = -6\times\log 10 - (-2x\times\log 10)$$

$$\Rightarrow \quad -4 = -6 + 2x \Rightarrow 2x = 6 - 4 = 2 \Rightarrow \quad x = 1$$

**4. (50)** $t_{1/2} = \frac{0.693}{K} = \frac{0.693}{0.0693} = 10 \text{ min}$

$t_{3/4} = 2\times t_{1/2} = 20 \text{ min}$

$t_{7/8} = 3\times t_{1/2} = 30 \text{ min}$

$t_{3/4} + t_{7/8} = 50 \text{ min}$

**5. (67)** Products of electrolysis of $aq.Na_2SO_4$ are $\underset{[\text{Cathode}]}{H_2(g)}$ and $\underset{[\text{Anode}]}{O_2(g)}$

equivalents of $H_{2(g)}$ and $O_{2(g)}$ produced will be same.

$\Rightarrow \quad \text{Equivalent} = \text{Moles} \times \text{n-factor}$

$\Rightarrow \quad \text{Moles} \propto \dfrac{1}{\text{n-factor}}$ [If equivalents are same]

$\Rightarrow \quad \dfrac{\text{Moles}(H_2)}{\text{Moles}(O_2)} = \dfrac{\text{n-factor}(O_2)}{\text{n-factor}(H_2)} = \dfrac{4}{2} = \dfrac{2}{1}$

Total moles of $H_2(g) \,\&\, O_2(g) = \dfrac{3.36}{22.4} = 0.15$

Moles of $H_2 = \dfrac{2}{2+1} \times 0.15 = 0.1$

$\text{Moles} \times \text{n-factor} = \dfrac{i \times t}{F} \times \dfrac{\%\ \text{efficiency}}{100}$ [Applying Faradays law for $H_2(g)$]

$0.1 \times 2 = \dfrac{9.65 \times t}{96500} \times \dfrac{50}{100}$

$t = 4000\ \text{s} = 66.67\ \text{min} \simeq 67\ \text{min}$

**6. (5)** $Pi = 10\ \text{atm}$

$P_f = (10 - 50\%\ \text{of}\ 10)\ \text{atm} = 5\ \text{atm}$

$|\Delta p| = 5\ \text{atm}$

$PV = nRT$

At const. 'V' and 'T', $P \propto n$

$\Rightarrow \quad \Delta P \propto \Delta n$

$\Rightarrow \quad \Delta P \times V = \Delta nRT$

$\Rightarrow \quad 5 \times 5 = \Delta n \times \dfrac{1}{12} \times 300 \quad \Rightarrow \quad \Delta n = 1$

Now moles of gas adsorbed = 1

Mass of gas adsorbed $= \text{Moles} \times \text{Molar Mass}$

$= 1 \times 30 = 30\ \text{g}$

Mass of gas adsorbed per unit gram of charcoal

$(x/m) = \dfrac{30}{60} = 0.5 = \cancel{Z} \Rightarrow 10\cancel{Z} = 5$

**7. (3)** Unit of $k = \text{mol}\ L^{-1}\ s^{-1}$

$\downarrow$

Zero order reaction

$\Rightarrow \quad x = k \cdot \times t$ (x – Conc. Of product at time t)

$\Rightarrow \quad 0.6 = k \cdot \times 10 \times 60$

$\Rightarrow \quad k \cdot = \dfrac{0.6}{600}\ \text{Mol}\ L^{-1}\ s^{-1} = 0.001\ \text{mol}\ L^{-1}\ s^{-1}$

$k \cdot = 0.001 = 1 \times 10^{-3} = 1 \times 10^{-x}$

$x = 3$

**8. (83)** $Ag \longrightarrow Ag^{+} + e^{-} \quad \Delta \overset{\circ}{G}_1 = -1 \times F \times (-0.80)$

$AgI + e^{-} \longrightarrow Ag + I^{-} \quad \Delta \overset{\circ}{G}_2 = -1 \times F \times x$

$AgI \longrightarrow Ag^{+} + I^{-} \quad \Delta \overset{\circ}{G}_3 = \Delta \overset{\circ}{G}_1 + \Delta \overset{\circ}{G}_2$

$\Delta \overset{\circ}{G}_3 = -2.303\,RT \log K_{SP} = F(0.80 - x)$

$\Rightarrow \quad -\frac{2.303\,RT}{F} \cdot \log KSP = 0.80 - x$

$\Rightarrow \quad -0.06 \times \log\left(8 \times 10^{-17}\right) = 0.80 - x$

$\Rightarrow \quad 0.966 = 0.80 - x$

$\Rightarrow \quad x = -0.166 \Rightarrow \left|\frac{1000\,x}{2}\right| = 83$

**9. (24)** $\frac{(\text{Flocculation value})x^{+}}{(\text{Flocculation value})y^{4+}} = \frac{4^6}{1^6}$

$\Rightarrow \quad \frac{100}{Z \times 10^{-3}} = 4096$

$\Rightarrow \quad Z = \frac{100000}{4096} = 24.414$

Rounding off $\rightarrow Z = 24$

**10. (12)** $\Delta_f \overset{\circ}{H}_{Reaction} = \sum \Delta_f \overset{\circ}{H}_{(Prod)} - \sum \Delta_f \overset{\circ}{H}_{(React)}$

$= 2 \times \Delta_f \overset{\circ}{H}\left(A_{(g)}\right) - 1 \times \Delta_f \overset{\circ}{H}\left(A_{2(g)}\right)$

$= 2 \times 40 - 1 \times 20 \quad \text{kJ mol}^{-1}$

$= \quad 60 \text{ kJ mol}^{-1}$

$\Delta_f H_{Reaction} = (E_a)_f - (E_a)_b$

$\Rightarrow \quad 60 = Ea_f - 60$

$\Rightarrow \quad Ea_f = 120 \text{ kJ mol}^{-1}$

$x = 120$

$x/10 = 12$

# MATHEMATICS

## SECTION-1

**1.(A)** $y+\frac{1}{y}=x \quad \Rightarrow y=\frac{x\pm\sqrt{x^2-4}}{2},\ y\in[1,\infty)$

$\Rightarrow \quad y=\frac{x+\sqrt{x^2-4}}{2}$

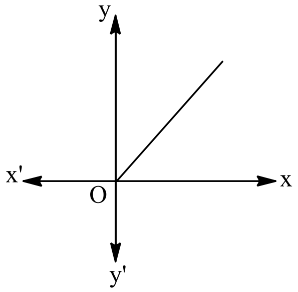
**2.(B)** The fundamental period of $|\sin x|+|\cos x|$ is $\frac{\pi}{2}$

**3.(B)** $\lim\limits_{x\to\pi^-/2}\left(\frac{1+\cos x}{\cos x}\right)a=$ finite only if $a=0$

$f\left(\frac{\pi}{2}\right)=b \Rightarrow b=a=0 \ ; \ \lim\limits_{x\to\pi^+/2}\frac{e^{-\cos x}-2c}{\sqrt{x-\frac{\pi}{2}}}=f\left(\frac{\pi}{2}\right)=0 \Rightarrow c=\frac{1}{2}$

**4.(A)** Given, $f(x)=x+|x|$

$\therefore \ f(x)=\begin{cases}2x, & x\ge 0\\ 0, & x<0\end{cases}$




It is clear from the graph of $f(x)$ is continuous for every value of $x$

Alternate

Since, $x$ and $|x|$ is continuous for every value of $x$, so their sum is also continous for every value of $x$

**5.(B)** In $\frac{1}{[x]}$ domain is $(-\infty,0)\cup[1,\infty)$ So, $\lim\limits_{x\to 0}\frac{1}{[x]}=\lim\limits_{x\to 0^-}\frac{1}{[x]}=-1$

**6.(C)** $y=\frac{(1+x)(1+x^2)....(1+x^{16})(1-x)}{(1-x)}$

$y=\frac{1-x^{32}}{1-x} \Rightarrow \ y-yx=1-x^{32}$

$\Rightarrow \quad y'-y-xy'=-32x^{31}$

$xy'+y-y'=32x^{31}$

$xy''+y'+y'-y''=32\times 31x^{30}$

Put $\quad x=-1 \ \Rightarrow \ 2(y'-y'')=32\times 31$

$\Rightarrow \ y'-y''=496$

**7.(C)** Let $f(x)=|x|$ & $g(x)=-|x| \Rightarrow$ Non-differentiable at $x=0$

Then $f+g=0 \Rightarrow$ Differentiable at $x=0$

$f.g=-|x|^2 \Rightarrow$ differentiable at $x=0$

**8.(C)** $\lim\limits_{x\to\infty} x\sin\left(\frac{2}{x}\right)=\lim\limits_{x\to\infty}\frac{\sin\left(\frac{2}{x}\right)}{\frac{1}{2}\left(\frac{2}{x}\right)}=2$

**9.(B)** $x\to\frac{1}{x}$

$2f\left(\frac{1}{x}\right)-f(x)=x+\frac{1}{x}$

Adding both $\Rightarrow$ $f(x)+f\left(\frac{1}{x}\right)=2\left(x+\frac{1}{x}\right)$

$\Rightarrow$ $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=2$

**10.(C)** $\frac{\{x\}+1-1}{1+\{x\}}=1-\frac{1}{1+\{x\}},$ $1+\{x\}\in[1,2) \Rightarrow \frac{1}{2}<\frac{1}{1+\{x\}}\le 1$

$\Rightarrow 0\le 1-\frac{1}{1+\{x\}}<\frac{1}{2}$

$\Rightarrow$ Range $\equiv\left[0,\frac{1}{2}\right)$

**11.(B)** $f(x)=\sin\left(\log\left(x+\sqrt{x^2+1}\right)\right);$

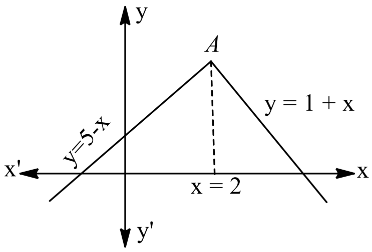
$$f(-x)=\sin\left(\log\left(-x+\sqrt{x^2+1}\right)\right)=\sin\left(\log\left(\frac{\left(\sqrt{x^2+1}\right)^2-x^2}{\sqrt{x^2+1}+x}\right)\right)=\sin\left(\log\left(\frac{1}{x+\sqrt{x^2+1}}\right)\right)$$

$=\sin\left(-\log\left(x+\sqrt{x^2+1}\right)\right)=-f(x)$ $[\because \sin(-x)=-\sin(x)]$ $\Rightarrow f(x)$ is an odd function

**12.(A)** Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)=\theta \Rightarrow \sin\theta=\frac{\sqrt{63}}{8}\Rightarrow \cos\theta=\frac{1}{8}$

$\Rightarrow$ $\sec(2\theta)=\frac{1}{2\cos^2\theta-1}=-\frac{32}{31}$

**13.(C)**




It is clear from the graph that $f(x)$ is continuous everywhere also it is differentiable everywhere except at $x = 2$.

**14.(C)** Let $\tan^{-1} a = \alpha, \quad \tan^{-1} b = \beta$

$\Rightarrow \quad \alpha + \beta = \dfrac{\pi}{4} \quad \Rightarrow \quad (1+\tan\alpha)\,(1+\tan\beta) = 2$

$\Rightarrow \quad (1+a)\,(1+b) = 2 \quad \Rightarrow (a,b)$ lies on a hyperbola

**15.(B)** $1 \le (\sin x + \cos x) \le \sqrt{2}$

$\Rightarrow \quad \cot^{-1}(1) \ge \cot^{-1}(\sin x + \cos x) \ge \cot^{-1}\left(\sqrt{2}\right)$

$f(x) \in \left[\cot^{-1}\sqrt{2}, \dfrac{\pi}{4}\right]$

**16.(B)** Let $f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & x \ne 0 \\ 0 & x = 0 \end{cases}$, $f(x)$ is differentiable at $x = 0$ but $f'(x)$ is not continuous at $x = 0$

**17.(B)** $\lim\limits_{x\to 0} \dfrac{e^{ax} - e^{bx}}{x} = \lim\limits_{x\to 0} \dfrac{ae^{ax} - be^{bx}}{1} = a - b$

**18.(A)** $S = \sum\limits_{r=0}^{n} \tan^{-1}\left(\dfrac{2^{r+1} - 2^r}{1 + 2^r \times 2^{r+1}}\right) = \sum\limits_{r=0}^{n} \tan^{-1}\left(2^{r+1}\right) - \tan^{-1}\left(2^r\right)$ telescopic series gives

$S = \tan^{-1}\left(2^{n+1}\right) - \dfrac{\pi}{4}$

**19.(A)** Take log & differentiate both sides
Put $x = 2024$

**20.(B)** $\tan^{-1}(x+1) + \tan^{-1}(x+2) = -\tan^{-1} x$

Apply $\tan \Rightarrow \dfrac{(x+1)+(x+2)}{1-(x+1)(x+2)} = -x$

Solving we get $x = \pm 1, 3$, but $x = 3, 1$ doesn't satisfy the original equation.

Hence only solution is $x = -1$

## SECTION-2

**1.(2)** $|x-3|\sin|x-3| = (x-3)\sin(x-3)$

$\cos(|x-3|)\left|x^2-6x+5\right| = \cos(x-3)\ |(x-5)(x-1)|$

$f(x) = (x-3)\sin(x-3) + \cos(x-3)|(x-5)(x-1)|$

Only N.D at $x = 5$ & $x = 1$

**2.(2)** Since, $\lim\limits_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2} = k \Rightarrow \lim\limits_{x\to 0}\dfrac{-(-\sin x)}{2x} = k$ [using L 'Hospital's rule]

$\Rightarrow \dfrac{1}{2}\lim\limits_{x\to 0}\dfrac{\sin x}{x} = k \Rightarrow \dfrac{1}{k} = 2$

**3.(7)** $2\log_{3/16}|\sin x| \le 1 - 2\log_{3/16}|\cos x|$

$2\log_{3/16}|\sin x\cos x| \le 1 \Rightarrow |\sin x\ \cos x| \ge \dfrac{\sqrt{3}}{4} \Rightarrow |\sin 2x| \ge \dfrac{\sqrt{3}}{2}$

$x \in [p, q] \cup [r, s]$; $\quad x \in \left[\dfrac{\pi}{6}, \dfrac{\pi}{3}\right] \cup \left[\dfrac{2\pi}{3}, \dfrac{5\pi}{6}\right]$

**4.(4)** $y = -2x^2 + 2x \Rightarrow$ Range of $g(x) = \left(-\infty, \dfrac{1}{2}\right]$

Range of $g(x)$ $\cap$ domain of $f(x)$

$\Rightarrow \left(-\infty, \dfrac{1}{2}\right] - \{-2\}$

Now, Range of $f(x)$ for $x \in \left(-\infty, \dfrac{1}{2}\right] - \{-2\}$ will be the range of $fog(x)$.

$y = f(x) = \dfrac{1+x}{2+x} \Rightarrow 2y + xy = x + 1 \Rightarrow x = \dfrac{2y-1}{1-y}$

C-I: $x \le \dfrac{1}{2} \Rightarrow \dfrac{2y-1}{1-y} \le \dfrac{1}{2} \Rightarrow y \in \left(-\infty, \dfrac{3}{5}\right] \cup (1, \infty)$

C-II: $x \ne -2 \Rightarrow \dfrac{2y-1}{1-y} \ne -2 \Rightarrow y \in R$

$\Rightarrow (C-I) \cap (C-II)$

$\Rightarrow$ Range of $fog(x) = \left(-\infty, \dfrac{3}{5}\right] \cup (1, \infty)$

**5.(0)** We have $f(x) = \sin[\pi]x + \sin[-\pi]x = \sin 3x + \sin(-4)x = \sin 3x - \sin 4x$

$\Rightarrow f(\pi) = \sin 3\pi - \sin 4\pi = 0$

**6.(4)** $\lim\limits_{n\to\infty}\left[n^3\left(1-\dfrac{1}{n}-\dfrac{1}{n^3}\right)\right]^{\frac{1}{3}} + n\alpha + \beta = n\left(1-\left(\dfrac{1}{n}+\dfrac{1}{n^3}\right)\right)^{\frac{1}{3}} + n\alpha + \beta = n\left\{1-\dfrac{1}{3}\left(\dfrac{1}{n}+\dfrac{1}{n^3}\right)\right\} + n\alpha + \beta$

$\left(n-\dfrac{1}{3}\right) + n\alpha + \beta = 0 \Rightarrow \alpha = -1,\ \beta = \dfrac{1}{3} \Rightarrow 3\left(\dfrac{1}{3}-(-1)\right) = 4$

**7.(4)** $\dfrac{6x^2+5x+1}{2x-1}>0$ & $-1\le\dfrac{2x^2-3x+4}{3x-5}\le 1$

$\Rightarrow \quad \dfrac{(2x+1)(3x+1)}{2x-1}>0$ & $\dfrac{2x^2-3x+4}{3x-5}\ge -1$ & $\dfrac{2x^2-3x+4}{3x-5}\le 1$

$\Rightarrow \quad x\in\left(\dfrac{-1}{2},\dfrac{-1}{3}\right)\cup\left(\dfrac{1}{2},\infty\right)$ & $\dfrac{2x^2-1}{3x-5}\ge 0$ & $\dfrac{2x^2-6x+9}{3x-4}\le 0$

$\Rightarrow \quad x\in\left(\dfrac{-1}{2},\dfrac{-1}{3}\right)\cup\left(\dfrac{1}{2},\infty\right) \qquad x\in\left[\dfrac{-1}{\sqrt{2}},\dfrac{1}{\sqrt{2}}\right]\cup\left(\dfrac{5}{3},0\right)$ & $x\in\left(-\infty,\dfrac{5}{3}\right)$

$\Rightarrow \quad x\in\left(\dfrac{-1}{2},\dfrac{-1}{3}\right)\cup\left(\dfrac{1}{2},\dfrac{1}{\sqrt{2}}\right]$

$\Rightarrow \quad \dfrac{18}{5}(\alpha^2+\beta^2+\gamma^2+\delta^2)=4$

**8.(1)** We have $x=\sin^{-1}(a^6+1)+\cos^{-1}(a^4+1)+\tan^{-1}(a^2+1)$

It is possible only when $a=0$

Thus, $x=\sin^{-1}(1)+\cos^{-1}+(1)+\tan^{-1}(1)$

$=\dfrac{\pi}{2}+0+\dfrac{\pi}{4}=\dfrac{3\pi}{4}$

Therefore, $\sin\left(x+\dfrac{\pi}{4}\right)-\cos\left(x+\dfrac{\pi}{4}\right)$

$=\sin\left(\dfrac{3\pi}{4}+\dfrac{\pi}{4}\right)-\cos\left(\dfrac{3\pi}{4}+\dfrac{\pi}{4}\right)=1$

**9.(3)** Since sum of the roots is –ve and product of the root is positive, both roots are negative. Thus *m* is a negative root.

Now, $\tan^{-1}(m)+\tan^{-1}\left(\dfrac{1}{m}\right)$

$=\tan^{-1}(m)-\pi+\cot^{-1}(m)\ =-\pi+\left(\tan^{-1}(m)+\cot^{-1}(m)\right)=-\pi+\dfrac{\pi}{2}=-\dfrac{\pi}{2}$

Clearly, $k=-1$ Hence, the value of $(k+4)$ is 3.

**10.(0)** We have,

$$f(x)=|x|^3=\begin{cases}x^3, & x\ge 0\\ -x^3, & x<0\end{cases}$$

$\therefore$ (LHD at $x=0$) $=\lim\limits_{x\to 0^-}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\dfrac{x^3}{x}=0$

and,

$\therefore$ (RHD at $x=0$) $=\lim\limits_{x\to 0^+}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\dfrac{x^3}{x}=0$

Clearly, (LHD at $x=0$) $=$ (RHD at $x=0$)

Hence, $f(x)$ is differentiable at $x=0$ and its derivative at $x=0$ is 0.